Children's Use of Everyday Mathematical Concepts to Describe, Argue and Negotiate Order of Turn

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In this paper expressions that children themselves use to describe relationships between phenomena in the world, is - when placing a mathematical gaze on them- viewed as everyday mathematical concepts. The aim of the paper is to illuminate children's use of everyday mathematics in their social interaction. More specifically, the aim is to show in detail how four- to five-year-olds use everyday mathematical concepts to describe, argue and negotiate order of turn, in this case in their interaction around a computer at a Swedish preschool. The case study is based on five 4 to 5 year old children's activities involving a computer at a municipal preschool in Sweden. The children's interaction around the computer was video recorded and analyzed in detail from a participant-oriented perspective on interactional conduct. The analysis shows that the children use various expressions that can be interpreted as everyday mathematical concepts as communicative cultural tools in their social interaction. Furthermore, the results show that the children have actual use for these concepts in their argumentation for order of turn, and that the concepts they use seem to be most sufficient in their argumentation in this situated activity. A conclusion is that the everyday mathematical concepts used in the analyzed activity can form a foundation for developing more formal mathematical concepts.

Keywords: early childhood education, everyday mathematical concepts, everyday mathematics, order of turn, social interaction

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Introduction and aim

In Swedish preschool practices, mathematics is seen as part of everyday life and closely incorporated into language. This kind of mathematics, which Ginsburg (2006) describes as children's everyday mathematics, and which is sometimes also referred to as pre-mathematics (Baroody, 2004; Freudenthal 1971), is very common in children's daily activities. Moreover, it is an indispensable precondition for higher-level mathematics and strongly contributes to forming the foundation for the further development, learning and use of mathematics later in life (Cross, Woods & Schweingruber, 2009; Duncan et al., 2007; Freudenthal, 1971).

However, everyday mathematics is often "hidden" in everyday activities and language, and is therefore not always recognized as mathematics, either by the children themselves or by those around them (e.g. peers, teachers, parents, researchers). From an educational perspective, where the object of learning as well as the teacher's participation is of importance for children's learning (Pramling Samuelsson & Asplund Carlsson, 2008), this is a didactical challenge. By placing a mathematical gaze onto the children's activity, and thereby recontextualizing an everyday practice into a mathematical practice (Dowling, 2013), this paper aims to illuminate children's use of everyday mathematics in their social interaction. Seen through a mathematical gaze, expressions used by children to describe relationships between phenomena in the world are viewed as everyday mathematical concepts (Björklund, 2007). A more specific aim is therefore to show in detail how four-to five-year-olds uses everyday mathematical concepts to describe, argue and negotiate order of turns in their interaction around the computer in a Swedish preschool. The reason for putting focus on this is based on a belief that concepts and expressions used by children in their own everyday activities are vital in their development of more explicit mathematical concepts (Björklund & Pramling, 2013; Johnsen Højnes, 2000; Vygotskij, 1987), and that we lack knowledge about how, in detail, children use everyday mathematical concepts about order of turn in their play.

The study was conducted within a sociocultural theoretical framework (Lave & Wenger, 1991; Rogoff, 2003; Säljö, 2000; Vygotskij, 1987), which implies that knowledge is viewed as situated and built up through social interaction (Säljö, 2005). Learning is seen as increasing participation in a socio-cultural practice, and as legitimate peripheral participation in a community of practice (Lave & Wenger, 1991). Lave and Wenger (1991) argue that the social and cultural practice in which one participates is both a breed-

ing ground for learning and a core part of what is learned. They see learning as participation in a social practice, where the actual learning consists of an increasingly developed participation. A fundamental idea is that knowledge is continuously created, used and transformed in the social organization as a whole (the community of practice). In a community of practice knowledge exists in the relationship between the participants, embedded in their practice and in its artifacts. This cultural and social knowledge is incorporated successively by the individual through an increasingly developed participation in the community – starting as a peripheral newcomer to eventually becoming a central and knowledgeable old-timer. Central to the theory is that learning is situated, which means that learning occurs in the context in which the knowledge belongs, is used and is beneficial. Knowledge is thus important for increasing participation in the practice, which means that knowledge is also coveted by the participants (Lave & Wenger, 1991).

The children's interaction and use of everyday mathematics are analyzed from a participant-oriented perspective on interactional conduct (Garfinkel, 1967). The analyses are based on members' perspectives, as revealed within talk-in-interaction and local activity frameworks. The ethnomethodological view on members' perspectives is consistent with a theoretical view on children's apprenticeship as situated and constituted through social interaction and accomplished through talk within a practice (Lave & Wenger, 1991).

Everyday mathematics in early childhood education

Throughout history mathematical knowledge has to a large extent developed from human activities in everyday life. According to Bishop (1991), the need for mathematical tools in humans' everyday lives has always been the driving force in developing new mathematical knowledge; knowledge that is preserved in language. Accordingly, in Swedish preschool practices mathematics is seen as closely incorporated into everyday life and language.

Early childhood mathematics has been focused on in research for a long time. Mostly the focus has been on how children develop understanding for mathematical ideas (e.g. Butterworth & Harris, 1994; Cross et al., 2009; Devlin, 1994; Piaget 1964, 1969; Sarama & Douglas, 2009). However, not that much research has focused on children's use of mathematics in their play and everyday activities. Even if the research is limited, some research can be found both in the Nordic countries and internationally (e.g. Björklund, 2007, 2008, 2010; Fauskanger, 1998; Flottorp, 2010; Ginsburg, 2006; Reis, 2011; Seo & Ginsburg, 2004).

In a case study Flottorp (2010) analyze an episode where two 5 year-old boys are playing in the sandpit. She argues that the children by creating structure and seeking meaning in their own classification of toys, are involved in a mathematical activity where mathematical order and structure become conscious experiences to the children. Flottorp (2010) emphasizes that the children are doing this as a part of their own play and not because some one has told them to. Ginsburg (2006) discusses the role of mathematics in children's play and the role of play in early mathematics education. He argues that everyday mathematics is a significant aspect of children's play, and that it not only provides the cognitive foundation for a good deal of play and other aspects of children's lives, but also often occurs as explicit mathematical content in children's play. Furthermore, Fauskanger (1998) emphasizes the roll of free play in learning mathematics. She argues that children use – and learn – mathematics in their free play by solving mathematical problems, using mathematical language (my stick is bigger than yours), and by creating and trying out rules in their play. Fauskanger also emphasizes the importance of children's dialogues in making a bridge between play and learning. In accordance with this, Björklund (2007) argues that preschool children seem to use everyday mathematics in mainly three areas, or in three ways. Firstly, they use it when solving mathematical problems. Young preschoolers often use everyday mathematics as a tool for problem-solving. For instance, children make use of different strategies when solving problems, e.g. a jigsaw puzzle. Logical thinking, whereby children are able to think beyond a present situation, is also an important aspect of both problem-solving and mathematical understanding. Secondly, children use everyday mathematics to create, test or abide by social rules. Abiding by social rules involves sharing tools equally among each other and taking into consideration other's skills and capacities. It also includes following everyday rules or routines, and seeing to it that everyone else does so as well. Everyday rules can involve the order in which things are to be done in a certain situation, for instance "lunch procedures" (first you get milk, then you can eat). Björklund (2007) asserts that young preschool children clearly express their understanding of such rules and the necessity for all to follow them. Thirdly, and perhaps most importantly, children use everyday mathematics as a linguistic tool. Preschool children use mathematics in their everyday activities to describe their surrounding world. In doing so, they use many different expressions and mathematical concepts (e.g. big, small, tall, taller, tallest, behind, over, under, before, after, etc.). In order to understand

and give meaning to these concepts, children continuously negotiate their understanding of them. When someone applies a concept differently, they often struggle to make that someone aware of their own understanding of this concept, or of their own understanding of which concept should be used to describe a specific phenomenon. This is also stressed by Bishop (1991), who argues that not only children but also adults frequently use everyday mathematics and mathematical concepts as a tool in arguing and explaining.

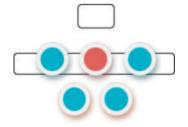
Thus, it is well known that children use everyday mathematics in their everyday activities in preschool. In this paper this is taken for granted and instead aims to show *how this is done in more detail*. More specifically it aims to contribute knowledge on how four- to five-year-olds use everyday mathematical concepts to describe, argue and negotiate order of turn in their interaction around the computer at a Swedish preschool.

Participants and setting

This article is based on a case study of five children's activities involving the computer. The case study emerges from a larger study with 22 children aged three to five years at a municipal preschool in Sweden. At this preschool, like at most preschools in Sweden, a *free play* period was scheduled on a daily basis. During this period the children were free to choose any activity they wanted without major interference from the teachers. The teachers had prepared the preschool environment for these kinds of activities, however; for example, the children could choose to use the one computer in the room to play any of the three available edutainment games.

In the excerpts shown below, five children participate: three girls (Hanna (5), Fia (5), Lisa (5)) and two boys (Elvis (5) and Botvid (4)). When the children used the computer during free play periods, they had to relate both to the *physical setting* around the computer and to the established *rules* regulating the computer use activity during free play.

Figure 1. Positions around the computer according to the established rules



According to the teachers, they had arranged the area of the room where the children used the computer based on educational and social considerations as well as physical limitations and regard for other activities in the room. A couch with room for three children was placed in front of the computer. The teachers' intention was to allow room near the computer for more than one child at a time. This was seen primarily as an educational benefit, but also as a way to give more than one child at a time the opportunity to be involved in using the computer.

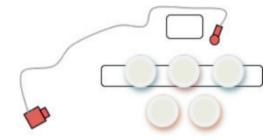
In addition to this physical arrangement the teachers had made rules regulating the computer use activity during free play. These rules had developed as a result of a situation in which the computer was highly desirable by the children on the one hand, and was a limited resource on the other. The rules stated that a maximum of three children were allowed to sit on the couch at once, and that no more than two children were allowed to stand behind the couch and watch (Figure 1). Furthermore, the children were to take turns using the computer, and each child had a limited time as the "owner" of the mouse (e.g. the player, Ljung-Djärf, 2008). The duration was regulated by a timer, set and managed by the children themselves. When the set time ran out, e.g. when the timer rang, the child who had been playing the game had to leave his or her place, allowing a shift of positions around the computer.

Data collection and analysis

Given the focus on how children use everyday mathematical concepts in their social interaction around the computer, the use of video recording as a tool for data collection was particularly relevant in this study. Since combined video/audio recording captures not only the verbal communication but also extensive additional information concerning other modes such as gestures and body movements, video recordings are especially suitable for analyzing different aspects of social interaction (Lantz-Andersson, 2009). A video camera placed on a tripod was used to video record the children's interaction around the computer. The camera was positioned to capture as much as possible of the children's visual interaction, most commonly at some angle behind and to the left of the computer (Figure 2). In order to capture the children's verbal interaction, an external microphone was connected to the camera and placed in front of the children beside the computer (Figure 2).

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Figure 2. Placement of video- and audio-recording devices in relation to the children and the computer.



The data were collected during two major periods: the first during two weeks in late spring, and the second during a three-week period the following autumn. In total, the researcher visited the preschool on 15 occasions, which rendered four and a half hours of film. Prior to the video observation of the children permission was obtained from the children's parents as well as from the children themselves. Initial informal observations and interviews with children and teachers were also performed. Since the video camera was placed on a tripod and generally managed itself, the researcher had the opportunity to act independently of the camera. During the first period of video recordings the researcher alternately took on the role of curious and inquisitive participant observer and that of a more passive observer, taking complementary field notes. As Cobb-Moore, Danby and Farrel (2009) argue, different researcher roles can be appropriate and successful in this kind of research, but considering that an adult most likely exerts at least some influence on the children's interaction in one way or another, the role of a passive observer could be preferable when an authentic everyday situation is the research focus (Carsaro & Schwarz, 1999; Mandell, 1991). Much for the above reason, during the second time period the researcher almost solely took on the role of passive note-taking observer, trying to minimize interaction with the children.

The analysis of the four and a half hours of film was conducted in three steps. First, a mathematical gaze was placed on the video recordings in order to find and illuminate any mathematical activity. Episodes that could be interpreted as containing any mathematical activity were separated and indexed with respect to main mathematical content. The indexing primarily derived from the researcher's interpretation of the observed situated interaction, but was also interpreted based on earlier research and the mathematical content standards of the US National Research Council (Cross et al., 2009). Second, episodes, or parts of episodes, containing interaction concerning order of turn were carefully transcribed using conversation analytic notation (cf. Hutchby & Wooffitt, 1998, see also Appendix A) and analyzed in detail from a participant-oriented perspective on interactional conduct (Schegloff, 1999). In the analysis the participant's own interpretation and understanding of what had been said or done earlier in the interaction was taken as base for a "correct" analysis from a participant-oriented perspective. This procedure, called next-turn proof procedure (Sacks, Schegloff & Jefferson, 1974), is commonly used in conversation analysis to ensure the relevance of the analysis. Third, a mathematical gaze was placed on the children's use of

different expressions for describing, arguing and negotiating order of turn, and called *everyday mathematical concepts*.

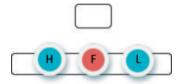
Analysis of the children's interaction

The following excerpts (1-4) illustrate parts of a longer sequence of the children's interaction around the computer during which order of turn is discussed. Each excerpt is analyzed separately in detail.

Excerpt 1 - Now it's my turn (Hanna (5), Elvis (5), Lisa (5), Researcher)

In Excerpt 1, three children are sitting on the couch in front of the computer. Elvis is the owner of the mouse while Lisa and Hanna are sitting beside him (Figure 3). When we enter the situation the timer has just rung, indicating a

Figure 3. Positions around the computer, H=Hanna, E= Elvis, L=Lisa.



shift of positions around the computer:

```
1
     Lisa:
                    nu är det min tur
                    now it's my turn
     Elvis:
                    nehe
                    no
3
     Lisa:
                    Hannas?
                    Hanna's?
4
     Elvis:
                    nej
5
     Res.:
                    jo, jag tror, titta
                    oh yes, I think so, look
6
                    (0.5) ((forskaren sträcker sig fram och
                    tar ned äggklockan och visar den för El-
                    vis och de andra))
                     ((the researcher leans forward and puts
                    the timer down, showing it to Elvis and
                     the others))
6
                     (0.5) ((forskaren sträcker sig fram och
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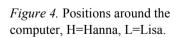
7	Res:	den är färdig
		it's done
8	Elvis:	jaha
		oh I see
9		(1)
10	Elvis:	jag ska bara göra en till övning
		I'm just going to do one more exercise
11	Lisa:	då gör (1) vi (xx) den här då
		then (1) we do this (xx) then
13	Elvis:	det är den sista då
		It's the last one then
14	Hanna:	ja
		yes
15	Elvis:	ja
		yes
16	Hanna:	jag är trea
		I'm number three
18	Elvis:	två be(x) vi ska välja <u>två</u> belöningar
		two be(x) we have to choose \underline{two} rewards
19	Lisa:	då får du göra <u>två</u> gånger till bara nu
		då
		you only get to do two more times now
		then
20	Elvis:	nej <u>en</u>
		no <u>one</u>
21	Hanna:	jag tycker att ettan skulle sitta <u>där</u>
		där Lisa sitter
		I think number one should sit there
		where Lisa's sitting
22	Hanna:	((Hanna pekar mot Lisa bakom ryggen på
		Elvis))
		((Hanna points at Lisa behind Elvis))
23	Elvis:	((fortsätter spela spelet under tyst-
		nad))
		((keeps playing the game in silence))

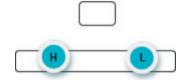
According to Lisa, on Line 1, it is her turn to play when the timer has rung. Elvis, who should move according to the set rules of turn-taking, contradicts Lisa's assertion. His answer can be interpreted as disagreement that it is Lisa's turn to play, but it could also mean that he is not ready to leave (Line 2). On Line 3 Lisa displays uncertainty about whether it is her turn. She therefore asks whether *Hanna* might be next in turn to play. Again, Elvis protests by saying "no" (Line 4). After an interruption by the researcher, Elvis agrees to leave after doing one more exercise (Lines 8-10), and states that it will be the last one (Line 13). Hanna then shows that she has come to the conclusion that she is *number three* in turn to play the game, after Elvis and Lisa, by saying "I'm number three" (Line 16). After Lisa and Elvis' short discussion on Lines 18-20, Hanna suddenly states that she thinks "... *number one* should sit...where Lisa's sitting" (Line 21). Her comment seems to emerge from nothing, and is also ignored by the others; at least no one acknowledges what she says (Line 23).

In the excerpt the children use everyday mathematical concepts to negotiate for order of turn in at least three situations (Lines 1, 16 and 21). On Line 1 Lisa uses the expression "now it's my turn" to declare her ordinal position in the order of turn. On Line 16 Hanna uses the concept of ordinal number by using the ordinal number word *number three* to describe her place in the order of turn, and on Line 21 she again uses the same form of ordinal number, when she states that she thinks "... that *number one* should sit...where Lisa's sitting". The use of ordinal numbers like *number one*, *number two*, *number three* instead of the more conventional notion of ordinal numbers - *first*, *second*, *third* - was very common among the children in this activity.

Excerpt 2 - I was number three (Hanna (5), Lisa (5))

In Excerpt 2 Elvis has just left his position as the owner of the mouse, leaving Hanna and Lisa alone on the couch (Figure 4):





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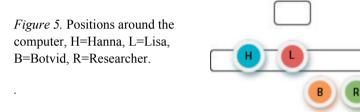
1	Lisa:	Hanna, din tur
		Hanna, your turn
2	Hanna:	men jag var ju
		but I was
3		(1.5)
4	Hanna:	trean
		number three
5	Lisa:	nej
		по
6		(1)
7	Lisa:	ok, då är det
		ok, then it's
8		(0.5)
9	Lisa:	jag
	2.54.	лч5 т е

On Line 1 Lisa offers Hanna the opportunity to be the owner of the mouse when Elvis has left. This can be interpreted as Lisa believes that Hanna actually is next in turn to play; seen as a whole, however, a more plausible interpretation is that Lisa is somewhat unsure of the order of turn, and offers Hanna the next turn. Hanna contradicts the offer, and makes it clear that she was number three in turn, which implies that according to her it is Lisa's turn to play (Lines 2-4). Finally, Lisa first refuses Hanna's offer (Line 5) but then accepts it and agrees to be the next player (Lines 7-9).

On Line 1 Lisa uses the expression "Hanna, your turn". This is a similar expression to the one she used earlier in Excerpt 1 (Line 1), and is common in arguing for order of turn among the children. On Lines 2-4 Hanna again uses an ordinal number, saying she was *number three* in turn, implying that it is Lisa's turn to play. It is notable that when Hanna says she *was* number three - using past tense - she is probably referring to the earlier situation and not the present one, in which she actually is number two after Lisa.

Excerpt 3 - I'm after her because I'm number three (Hanna (5), Botvid (4), Researcher)

In Excerpt 3 Lisa is the new owner of the mouse, playing the game. Hanna is sitting beside her on the couch when Botvid comes along. The researcher is



also standing nearby (Figure 5):

1	Botvid	är det min tur snart
		is it my turn soon
2		(3)
3	Res:	nej nu spelar Lisa tror jag
		no, I think Lisa's playing now
4	Botvid:	du
		you
5		(1)
6	Botvid:	du menar Botvid
		you mean Botvid
7	Hanna:	nej jag är efter henne för jag är trean
		no, I'm after her because I'm number
		three

When Botvid asks if it is his turn soon on Line 1, he is very likely referring to the owner position. The question seems to be addressed to one and all, including the researcher standing nearby, and indicates that Botvid actually does not know his possible position in the order of turn and wants an answer. When neither of the two girls answers him, the researcher steps in and clarifies that (he believes) Lisa has the owner position for the moment. Botvid's next utterance can be interpreted in several ways, but given the interaction to come, a possible interpretation is that he means that he himself is next in turn

after Lisa. Hanna's utterance on Line 7 can be seen as a response to either Botvid's first question on Line 1 or to his more unclear one on Lines 4-6.

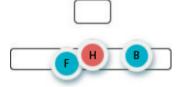
On Line 1 Botvid uses the same type of expression Lisa used at the start of the earlier excerpts when arguing for order of turn (Line 1 in Excerpts 1 and 2, respectively). He asks if it is *his turn* soon. On Line 7 Hanna uses two different everyday mathematical concepts in arguing for her ordinal position. She says "I'm *after* her..." which is a relational way to position herself in the order of turn. On the same line she also uses the ordinal number words *number three*, saying "... I'm number three".

What is of special interest here, however, is the way Hanna justifies her place in the order of turn. Hanna says she is *after* Lisa, the present owner of the mouse, *because* she, Hanna, *is number three*. Here Hanna first points out that she is *after* Lisa, the present player in turn, indicating that she now is in fact number two in turn to play. What is noteworthy is that she builds this statement on the fact that she *is number three*.

Excerpt 4 - Two different kinds of argumentation (Fia (5), Hanna (5))

In Excerpt 4 Hanna is the owner of the mouse and Botvid is sitting beside her, watching. When we enter the situation, Fia has just appeared and sits down beside Hanna on the couch (Figure 6). Of importance for understanding the excerpt is also that Fia, in a conversation with the researcher (Max),

Figure 6. Positions around the computer, F=Fia, H=Hanna, B=Botvid.



has just gotten the impression that it is now her turn to play on the computer:

1 Fia: hördu det är min tur

hey, It's my turn

2 Hanna: <va ↓sa du>

<What did you say>

3 Fia: det är min tur

It's my turn

4	Hanna:	(4) ((Hanna höjer ögonbrynen och
		tittar länge på Fia under tyst-
		nad))
		((Hanna raises her eyebrows and
		looks at Fia in silence for a
		long time))
5	Fia:	ge mig musen
		Give me the mouse
6	Hanna:	<u>nä</u>
		<u>no</u>
7	Fia:	<u>JO</u>
		<u>YES</u>
8	Hanna:	nej, jag var efter Lisa
		no, I was after Lisa
9	Fia:	men
		but
10		(1)
11	Hanna:	för jag var trean
		because I was number three
12	Fia:	jamen
		but (1) I
13		(1)
14	Fia:	Max sa att
		Max said that
15		(1)((Fia lutar sig fram mot Hannas öra))
		((Fia leans towards Hanna's ear))
16	Fia:	ojag fick spelao
		°I could play°
		_ ·

On Line 1 Fia shows up near the computer, claiming it is her turn to play. This statement seems to surprise Hanna, who replies, "What did you <u>say</u>" (Line 2). Fia repeats her demand, telling Hanna that it is her (Fia's) turn to play. Hanna replies by looking at Fia in silence for a long time, raising her eyebrows and showing surprise. Fia intensifies her demands, now telling Hanna to hand over the mouse (Line 5), which leads to another interchange of disagreement between the girls (Lines 6-7). On Line 8 Hanna again says no and adds an explanation for her claim, saying she (Hanna) "was after

Lisa" in turn, which implies that Hanna now has to be the player since Lisa has just given up ownership of the mouse. Fia tries to oppose Hanna's explanation but is interrupted by Hanna, who completes her argumentation by stating that she "was number three" (Line 11). Hanna is most certainly referring to the earlier event (Excerpt 1), something that is not obvious to Fia. On Lines 12-16, Fia finally asserts her argument for her turn, based on what she has heard from the researcher (Max).

In this excerpt the children use three different everyday mathematical concepts to argue for their position in the order of turn. On Line 1 Fia says "Hey, it's my turn" using the same type of expression used at the beginning of the earlier excerpts. On Line 8 Hanna says "no, I was after Lisa" using the relational word after to position herself in the order of turn. On Line 11 Hanna strengthens her statement that she was after Lisa in turn, using an ordinal number word – "because I was number three". Hanna's remark on Line 11 follows the same pattern as in earlier excerpts. On Lines 12-16, however, Fia declares that Max has said she could play. Her argument is expressed with some reservation, almost whispered, but at the same time has a strong impact because she refers to an adult. As Maynard (1985) asserts, adult-formulated arguments are often used by children to increase their own power within peer interactions, of which this is an illustration. However, the fact that Lisa utters her argument with some reservation indicates that she finds Hanna's argument convincing.

In summary, this excerpt shows how Hanna and Fia argue about who is number one in the order of turn, using two different kinds of argumentation. Hanna argues by referring to the proper order of turn in the abstract queue. Her arguments are based on her assessment of an earlier situation and she uses ordinal number words as a tool in her argumentation. Throughout the whole play event she has claimed that she is *number three*. Fia, on her part, refers to what she has heard from *an adult*.

Everyday mathematical concepts - sufficient and of actual use

The analysis of the excerpts shows that the children use various expressions that can be interpreted as everyday mathematical concepts as communicative cultural tools in their social interaction. Furthermore, the analysis shows that the children have actual use for these concepts in their argumentation and negotiation for order of turn, and that the concepts they use seem to be most sufficient in their argumentation in this situated activity.

Expressions used as everyday mathematical concepts

The children mainly used three different types of expressions to position themselves in the order of turn. One of the most frequent was the ordinal number words. Here the concepts used are numerical words. In negotiating and arguing for her position. Hanna often use concepts like number one and number three to position herself or someone else in the order of turn (I'm number three, Excerpt 1, Line 16; I think number one should sit where Lisa's sitting, Excerpt 1, Line 21; But I was number three, Excerpt 2, Lines 2-4; ...because I was number three. Excerpt 3. Line 7: Because I was number three, Excerpt 4, Line 11). When whole numbers (1, 2, 3...) are used to put items in order or in sequence, they are called *ordinal numbers*. First, second, and third (1st, 2nd, and 3rd) are typical notations of ordinal numbers, even if other notations can also be seen (Samara & Clements, 2009). Used in talk, such words would be labeled ordinal number words (Fuson, 1988). The common use of ordinal numbers like number one, number two, number three instead of the more conventional notion of ordinal numbers – first, second, third - is notable. I would argue that in their own activities children use words and expressions that are familiar and have meaning for them (Vygotskij, 1987). In this case the everyday mathematical concept number three has a meaning, whereas third might not. As Samara and Clements (2009) argue, it is a limited view to reserve only the notions of first, second, third and so on for use as ordinal numbers. A person who, for example, is number three in a line is labeled with a word that is no less ordinal in its meaning just because it is not expressed as third.

Another common expression used is when the children express that it is their (or someone else's) *turn* (*Now it's my turn*, Excerpt 1, Line 1; *Hanna, your turn*, Excerpt 2, Line 1; *Is it my turn soon*, Excerpt 3, Line 1; *Hey, it's my turn*, Excerpt 4, Line 1; *It's my turn*, Excerpt 4, Line 3). Here "my turn" refers to the first position in the sequence, in this case the position of the player or the owner of the mouse. This expression seems to be very useful in the children's activity - in any case if it concerns the first position. It is a concept that the children seem to be familiar with and to understand the meaning of.

On some occasions the children use concepts that express spatial or temporal relations to argue for order of turn. In both cases in which this is done, the word *after* is used (*No, I'm after her...*, Excerpt 3, Line 7; *No, I was after Lisa*, Excerpt 4, Line 8). The children argue by stating their position directly in relation to someone else – "after her" or "after Lisa".

One could of course question if the analyzed expressions used by the children could really be seen as everyday mathematical concepts. I argue that it depends on the situated practice and the perspective from which it is seen. Seen from the participating children's own perspective they probably wouldn't see their expressions as mathematical. Some people might argue that, regardless of context, the concepts labeled ordinal number words would be seen as mathematical concepts while the my-turn expression would not, and that it would be a question of context as to whether or not the concepts expressing spatial or temporal relations were to be seen as a mathematical or not. I argue that, provided a mathematical gaze is placed on the *concept of* order of turn, all expressions used by the children to describe, argue and negotiate order of turn should be seen as everyday mathematical concepts. The expressions they use are their own expressions in that that they know and understand the meaning of them. Since the importance of taking departure in expressions that are familiar and known by the children in developing an understanding of more nuanced mathematical concepts is often stressed (e.g. Björklund & Pramling, 2013; Johnsen Højnes, 2000; Vygotskij, 1987). the everyday mathematical concepts used by the children have the potential to develop into more formal mathematical concepts.

Of actual use in argumentation for order of turn

It is prominent that the concepts the children use truly prove to be of actual use in their argumentation. The children use everyday mathematical concepts not only as communicative cultural tools in general (Säljö, 2000, 2005), but also as useful tools in their argumentation for order of turn. The way Hanna uses ordinal number words in the excerpts above is an illustration of this. Throughout the negotiation in Excerpts 1-4, she uses ordinal number words and claims that she is (or was) after Lisa because she is (or was) number three. In Excerpts 2 and 4 she refers to her position, number three, in past tense ("I was number three"), which implies that she is referring to the earlier situation in Excerpt 1. In Excerpt 1 number three was a proper ordinal number notation, and as she refers to that situation in Excerpts 2 and 4, number three must be seen as an ordinal number in these situations as well. The reason Hanna refers to the earlier situation in Excerpt 1 in Excerpts 2 and 4 (where she is actually number two and number one in turn, respectively) is most probably not because she might find it hard to use ordinal numbers, but more likely because she is using this as an argument in her claim regarding her present position. By making an assessment of a situation in which, according to herself, her

position as number three was undoubtedly true, she establishes a solid foundation for her argument. Saving "It's my turn now, because I was number three [in the earlier situation]" is – provided that Fia shares Hanna's experience of the situation in question – undoubtedly a better way to argue than saying "It's my turn now because I'm number one". In Excerpt 3 Hanna argues for her position as number two by saying "I'm after her because I'm number three". Most probably, Hanna has the same line of argument in mind here as in Excerpts 2 and 4. The fact that she says "I am number three" does not necessarily make any difference. However, her utterance could be understood as using the words as a label for her identity as number three - "I'm number three", as in for example "I'm Batman". Using numbers as a label or identity in this and similar ways is common among children who have difficulties with number sense (Doverborg, 1985; Sinclair, Siegrist & Sinclair, 1983; Sterner & Johansson, 2006). However, what is illustrated by Hanna's argumentation is how everyday mathematical concepts are of actual use in her argumentation for order of turn.

Excerpt 4 shows how Hanna and Fia argue about who is number one in the order of turn using two different kinds of argumentation. Hanna argues by referring to the proper order of turn in the abstract queue. She argues based on an earlier situation, and uses ordinal number words as a tool in her argumentation. Throughout the whole activity she has claimed that she is *number three*. Fia, on her part, refers to what she has heard from *an adult*. Hanna uses a (pre-) mathematical argumentation based on mathematical logic while Fia does not.

Sufficient in the situated activity

As shown, the children used the type of words described above in arguing for ordinal position. Not only expressions like "I'm after her" and "It's my turn" but also expressions containing ordinal number words like "I'm number three" are used to argue for their ordinal position. These words and expressions can perhaps be regarded as not fully sufficient for describing an ordinal position. However, in the frames of this specific situated activity, the words used prove to be most sufficient in arguing for ordinal position. Again, the expressions used by the children are their own – expressions they know the meaning of in this situated activity.

Discussion

The aim of this paper is to illuminate children's use of everyday mathematics in their social interaction. More specifically, the aim is to show in detail how four- to five-year-olds use everyday mathematical concepts to describe. argue and negotiate order of turn in their interaction around the computer at a Swedish preschool. A main reason for this is the assumption that children's everyday activities contain events and communication impregnated with "hidden mathematics". Furthermore, this so-called everyday mathematics (Ginsburg, 2006) or pre-mathematics (Baroody, 2004; Freudenthal 1971) is seen as an indispensable precondition for higher-level mathematics and strongly contributes to forming the foundation for the further development, learning and use of mathematics later in life (Cross et al., 2009; Duncan et al., 2007; Freudenthal, 1971). By analyzing in detail children's interaction from a participant-oriented perspective (Schegloff, 1999), combined with placing a mathematical gaze (Dowling, 2013) on the children's everyday activity, a detailed description of how children use everyday mathematical concepts to describe, argue and negotiate order of turn in a situated activity is made possible. But how can this knowledge be of educational use? How can this activity help children learn something?

Based on a socio-cultural theory on learning, people always learn something by participating in a situated activity. The question of *what* has been learned is however not always that easy to answer. Seeing the situated activity as an event of potential learning, where the children's desire for participation direct their learning, could give a hint of what they might learn from the analyzed situations (Bevemyr & Björk-Willén, forthcoming).

In the context in which the children participate, the maintenance of a proper order of turn on which everyone can agree is, from the children's perspective, the essence of the activity. Being involved in this context is seen as important for the children, and knowledge in describing, arguing and negotiating order of turn is therefore seen as essential and desirable. In other words, from a situated learning theoretical point of view (Lave & Wenger, 1991) there is reason to believe that the situated activity in which the children are engaged has potential for helping them learn the situated knowledge required for the maintenance of a proper order of turn on which everyone can agree – situated knowledge including describing, arguing and negotiating order of turn.

In a mathematical educational context, though, one could argue that seeing the children's activity as situated learning in a community of practice

would not at all, or just briefly, contribute to their learning of formal mathematics. The children might learn, and even become experts in, arguing and negotiating for order of turn, but they would not learn formal mathematics or formal mathematical concepts. To make this happen, it is vital that a mathematical gaze be placed on the situation – and that this be done by the preschool teachers. Preschool teachers have an outstanding opportunity to help children relate the concepts used in the situated activity to concepts used in other situations as well as to more formal mathematical concepts. Using children's own expressions and everyday mathematical concepts as a starting point, however, is an important step in this process (Björklund & Pramling, 2013; Johnsen Højnes, 2000).

This paper gives a detailed analysis of how four- to five-year-olds use everyday mathematical concepts to describe, argue and negotiate order of turn in their interaction around the computer at a Swedish preschool. A conclusion that can be made is that the everyday mathematical concepts used in the analyzed activity have the potential to form a foundation for developing more formal mathematical concepts (Vygotskij, 1987). Hopefully, it will inspire preschool teachers to place a mathematical gaze on children's everyday activities, illuminate the "hidden mathematics", and help children develop formal mathematical concepts in similar preschool activities.

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Children's Use of Everyday Mathematical Concepts to Describe, Argue and Negotiate Order of Turn

Appendix A

text

[]	Square brackets mark the start and termination of overlapping speech.	
Underlining	1	
CAPITALS	Capital letters mark speech that is obviously louder than sur-	
	rounding speech.	
•	Quieter speech	
↓ 1	Indicates falling intonation in succeeding syllable(s).	
(2)	Pauses measured in seconds	
((text))	Transcribers' comments	
> word<	Quicker than surrounding speech	
<word></word>	Slower than surrounding speech	
(x)(xx)	naudible word or words	
text	Swedish	

Translation into English